



ENVIRONMENTAL IMPACT ON THE SPREAD OF DENGUE VIRUS WHEN TWO MOSQUITO SPECIES CIRCULATE

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Abstract

We study the environmental impact on the dengue fever epidemic

Received: June 6, 2016; Revised: July 5, 2016; Accepted: July 23, 2016

2010 Mathematics Subject Classification: 97Mxx.

Keywords and phrases: *Aedes*, dengue infection, endemic equilibrium points, limit cycle, temperature.

when two species of mosquitoes namely the *Aedes aegypti* and *Aedes albopictus* circulate. The infected populations of the humans and of mosquito species are each divided into two groups, the being infected but not infectious and the other infected and infectious (I). A SEIR model and SEI model are developed for the human and the two mosquito populations, respectively. The dynamical behaviors of the three populations are obtained by simulating the behaviors by numerically solving the differential equations which describe the models. Changes in the trajectories when the values of the environment dependent parameters are changed reveal the influence of the environment on the spread of the dengue virus. Often the values of the parameters in the two SEI models will depend on how a particular mosquito specie interacts with the environment and so the values of the parameters will be different in the two SEI models. This makes possible that the basic production numbers of the two species ($S_{aegypti}$ and $S_{albopictus}$) will be different giving different S_i 's which give the following combinations ($S_{aegypti} > 1, S_{albopictus} > 1$), ($S_{aegypti} > 1, S_{albopictus} < 1$), ($S_{aegypti} < 1, S_{albopictus} > 1$) and ($S_{aegypti} < 1, S_{albopictus} < 1$). When the different conditions are satisfied, nature will have both species present, only one specie present or no species present.

1. Introduction

Dengue fever (DF) is one of the arthropod-borne diseases that is endemic in more than 100 tropical and subtropical countries mostly in Africa, Central America and Southeast Asia [1]. It can also infect countries in southern Europe, in particular, Italy and Spain. The World Health Organization (WHO) has labeled DF as being the most important vector borne disease since there are more than 2.5 billion people at risk to this disease [2]. The disease is caused by any one of the four serotypes of the dengue virus, labeled DEN1, DEN2, DEN3 and DEN4. Infection by one serotype does not lead to immunity to infections by another type of the virus [3].

The virus is spread to a human by the bite of two species of the *Aedes* mosquitoes, the *A. aegypti* and the *A. albopictus* mosquitoes. The discovery

that DF can be spread by the latter species has caused concern among the public health officials around the world since the *A. albopictus* mosquito can survive at colder temperatures [4], making it possible for DF to appear in the northern Europe and the United States (USA). Understanding the spread of this disease is of special importance to Thailand since epidemics involving three or more different serotypes of the dengue virus have occurred in the country. In most countries, the epidemic is due to only one of the serotypes [5]. Furthermore, both the *A. aegypti* and *A. albopictus* mosquitoes exist in Thailand [6]. The *A. aegypti* mosquitoes are found in the urban areas since the females lay their eggs in manmade containers such as discarded tires, cans or broken bottles. The *A. albopictus* mosquitoes are found in the rural areas since the females lay their eggs in natural crevices between the individual banana or bamboo trees which grow in clusters in the clearing in the jungle. The *A. albopictus* mosquito has however adapted itself so that it can also survive in the urban environment.

The females of both species lay their eggs above the water line of the breeding pool [7]. Therefore the eggs will only hatch when the eggs are inundated by water. In Thailand and other countries in South East Asia, this happens only after a heavy rainfall. In dry area such as in southwest United States, the water can come from the human watering of the crops [8]. Without the inundating water, the eggs will not hatch and there will be no more adult mosquitoes until the next rainy season. If there are no mosquitoes in the environment, there would no further infection humans. There should therefore be a correlation between the number of dengue infection and the start of the rain fall. In Figure 1 which is a plot of the number of cases of dengue fever in each month, the mean temperature and the cumulative rainfall in southern Thailand in 2010 [9], we do indeed see that the highest incidence of dengue infections occurs during the rainy season.

The temperature also affects the development of the dengue virus. The virus develops inside the mosquitoes in hot weather. But as the temperature decreases, the incubation period increases. At 16°C, the incubation period can be longer than the life time of the mosquitoes. Thus the mosquito will die before it becomes infectious. After the last rainfall of the season, the eggs

laid do not shrivel up and die since the eggs of both species at this time are desiccation resistant and survive for a long period during the dry season (between 3 to 8 months for the *A. aegypti* eggs [10, 11] and 340 days for the *A. albopictus* eggs [12]). This long desiccation resistance time for the *A. aegypti* eggs made it possible for the *A. aegypti* mosquitoes to be reintroduced into Cuba when used tires carrying the eggs were imported into Cuba from SE Asia [13]. Harrington et al., have forecast the effects of temperature and rain fall on the incidence of dengue fever [14, 15].

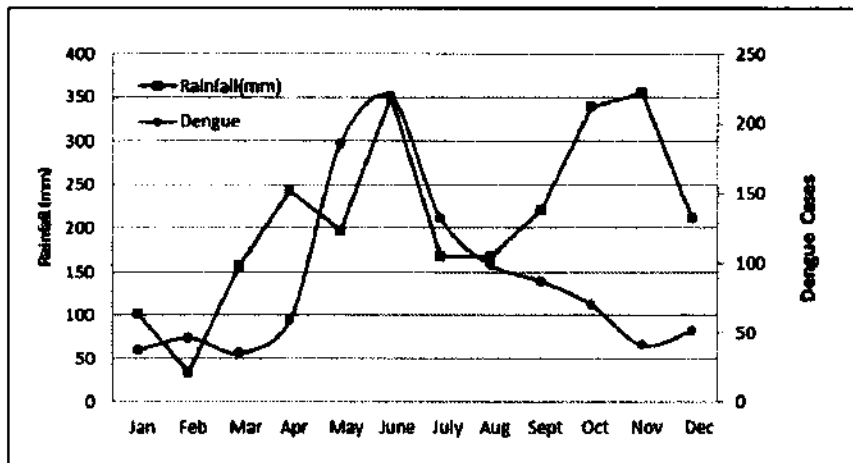


Figure 1. Reported cases of dengue cases, mean temperature and rainfall southern Thailand in 2010.

Other climatic effects that affect the survival of the different species have been reported by Juliano et al. [16]. They reported that the proportion of breeding sites occupied by the *A. albopictus* eggs at the start of the rainy season was significantly lower than the proportions well into the rainy season. This difference in the number of breeding sites occupied by the *A. aegypti* eggs did not appear to be affected by whether one was at the beginning or middle (or end) of the rainy season. This could be the reason for the seasonal shift in the relative abundance of the *A. aegypti* and the *A. albopictus* mosquitoes observed by Mogi et al. [6] in Chiang Mai, northern Thailand. The *A. aegypti* mosquitoes being the most abundant at the beginning of the rainy season and the *A. albopictus* mosquitoes being the

more abundant towards the end of the rainy season. Esteva and Vargas [17] were the first to develop a model for dengue disease transmission and which incorporated the dynamics of the *Aedes aegypti* mosquito into a standard SIR (S-susceptible, I-infected, R-recovered) model for the human population and SI model for vector. Pongsumpun and Kongnuy [18] utilized the Lyapunov function to determine the effect of maternal antibodies on the dengue disease in newly born infants. In order to take into account the fact that the dengue viruses need to incubate inside the human and mosquitoes before they can be transmitted to a different host, the infected class needs to be divided into an infected but not infectious class and an infectious class, the I class in the original model should be replaced by an E (exposed) and an I (Infectious) class. Thus the SIR model becomes a SEIR model while the SI model for the mosquitoes becomes a SEI model. If there are more than one serotypes of the dengue virus around, the recovered class can become a new susceptible class which can be infected by the second serotype. For dengue virus, the infection by one serotype confers immunity to human to that serotype but not to the other serotypes. Sriprom et al. [19] have developed a SEIRS model to study the destabilization effect of the host immune status on the sequential transmission dynamics of the DF infection.

For this study of dengue infection when there are two species of mosquitoes present, we use the SIERS model to describe the dynamics of the DF virus in the humans and two SEI models to describe the dynamics in the two species of mosquitoes. In this research, we use the data of dengue disease in southern Thailand, 2010 to create the model. Analysis of the model shows that there is a reproductive ratio S_0 that conceptualizes the rate of spread of a dengue disease and determines a threshold: when $S_0 < 1$, a typical infection gives rise, on average, to less than one secondary infection, and the disease will die out. When $S_0 > 1$, the disease will become endemic.

2. Formulation of Model for Dengue Disease

Sungchakit et al. [20, 21] used a SIR model for describing the dengue transmission by *Aedes aegypti* and *Aedes albopictus*. Since we are interested

in the effects of environment factors on the spread of DF in Thailand, we have used the SEIRS (host (human) population (the recovered can be susceptible to infection by another serotypes of the DENV) model to describe the transmission of the DF in Thailand. We have taken the mean temperature, and cumulative rainfall to be the values for those southern Thailand. In the flow chart appearing in Figure 2, $N_{a(b)}$ is the total human population which will be bitten by an *A. aegypti* mosquito (denoted by the subscript (*a*)) or by an *A. albopictus* mosquito (now denoted by the subscript (*b*)). The two human populations are each divided into five compartments, $S_i(t)$, $E_i(t)$, $I_i(t)$, $R_i(t)$ and $S'_i(t)$ are the numbers of humans who are susceptible to the dengue virus carried by a specie '*i*' mosquito (*i* is "*a*" if the specie is *A. aegypti* and is "*b*" if the specie is *A. albopictus* the *i*-th specie of mosquitoes, infected but not infectious, infectious, recovered and susceptible to infection by the second serotype virus, respectively. Denoting the total numbers of *A. aegypti* mosquitoes and *A. albopictus* mosquitoes as N_{va} or N_{vb} . If a second serotype virus is present, then the recovered humans would be susceptible to infection by the second serotype virus. The last group however is immune to further infections by the same serotype of virus. The populations of each species of the mosquitoes are separated into three subclasses, S_{vi} (susceptible '*i*' specie mosquito), E_{vi} (infected but not infectious '*i*' specie mosquito) and I_{vi} (infectious '*i*' specie mosquito). The flow charts for the different classes are shown in Figure 2.

The dynamical equations for the different population are obtained by inspection and are

$$\frac{dS_a(t)}{dt} = aN_h - \left(T_{va}\Psi_a\gamma_{vah} \frac{I_{va}(t)}{N_h + b} + \delta_h \right) S_a(t) + \theta_a R_a(t), \quad (1)$$

$$\frac{dE_a(t)}{dt} = \left(T_{va}\Psi_a\gamma_{vah} \frac{I_{va}(t)}{N_h + b} \right) S_a(t) - (\delta_h + \alpha_a) E_a(t), \quad (2)$$

$$\frac{dI_a(t)}{dt} = \alpha_a E_a(t) - (\delta_h + \beta_a) I_a(t), \quad (3)$$

$$\frac{dR_a(t)}{dt} = \beta_a I_a(t) - (\delta_h + \theta_a) R_a(t), \tag{4}$$

$$\frac{dS_b(t)}{dt} = (1-a)N_h - \left(T_{vb}\Psi_b\gamma_{vbb} \frac{I_{vb}(t)}{N_h + b} + \delta_h \right) S_b(t) + \theta_b R_b(t), \tag{5}$$

$$\frac{dE_b(t)}{dt} = \left(T_{vb}\Psi_b\gamma_{vbb} \frac{I_{vb}(t)}{N_h + b} \right) S_b(t) - (\delta_h + \alpha_b) E_b(t), \tag{6}$$

$$\frac{dI_b(t)}{dt} = \alpha_b E_b(t) - (\delta_h + \beta_b) I_b(t), \tag{7}$$

$$\frac{dR_b(t)}{dt} = \beta_b I_b(t) - (\delta_h + \theta_b) R_b(t). \tag{8}$$

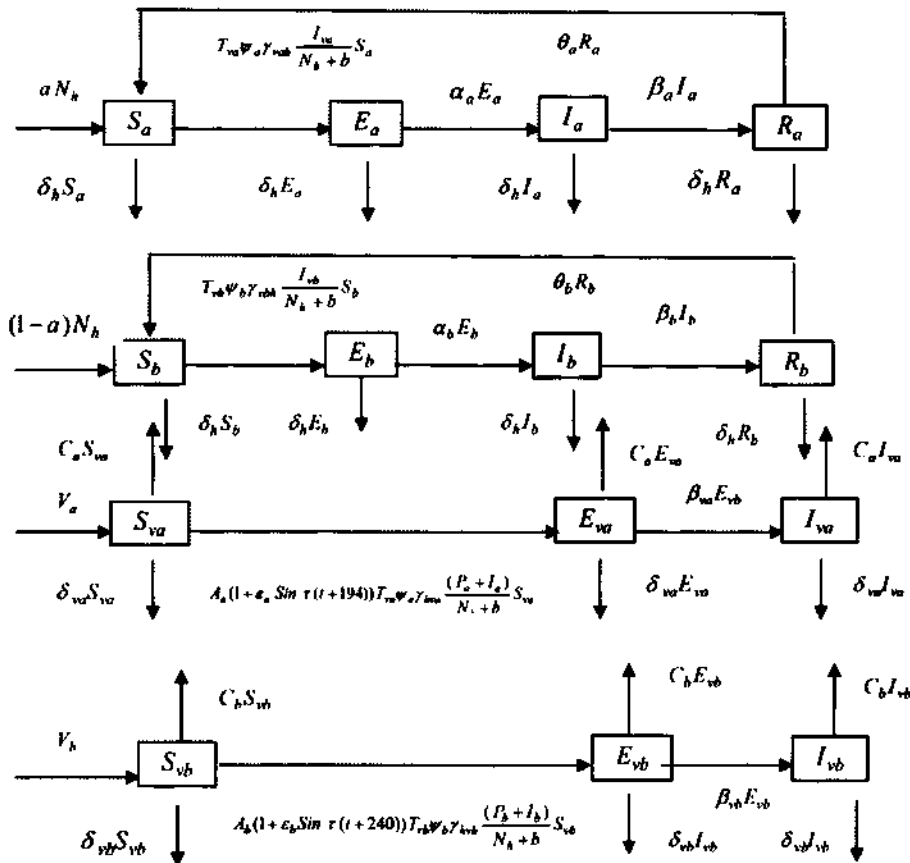


Figure 2. Flow chart of the model.

The dynamical systems are described by the following equations:

$$\begin{aligned} \frac{dS_{va}(t)}{dt} &= V_a \\ &- \left(A_a(1 + \varepsilon_a \text{Sin}\tau(t + 194)) T_{va} \Psi_a \gamma_{hva} \frac{(P_a + I_a(t))}{N_h + b} + C_a + \delta_{va} \right) S_{va}(t), \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{dE_{va}(t)}{dt} &= \left(A_a(1 + \varepsilon_a \text{Sin}\tau(t + 194)) T_{va} \Psi_a \gamma_{hva} \frac{(P_a + I_a(t))}{N_h + b} \right) S_{va}(t) \\ &- (C_a + \delta_{va} - \beta_{va}) E_{va}(t), \end{aligned} \quad (10)$$

$$\frac{dI_{va}(t)}{dt} = \beta_{va} E_{va}(t) - (C_a + \delta_{va}) I_{va}(t), \quad (11)$$

$$\begin{aligned} \frac{dS_{vb}(t)}{dt} &= V_b \\ &- \left(A_b(1 + \varepsilon_b \text{Sin}\tau(t + 240)) T_{vb} \Psi_b \gamma_{hvb} \frac{(P_b + I_b(t))}{N_h + b} + C_b + \delta_{vb} \right) S_{vb}(t), \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{dE_{vb}(t)}{dt} &= \left(A_b(1 + \varepsilon_b \text{Sin}\tau(t + 240)) T_{vb} \Psi_b \gamma_{hvb} \frac{(P_b + I_b(t))}{N_h + b} \right) S_{vb}(t) \\ &- (C_b + \delta_{vb} - \beta_{vb}) E_{vb}(t), \end{aligned} \quad (13)$$

$$\frac{dI_{vb}(t)}{dt} = \beta_{vb} E_{vb}(t) - (C_b + \delta_{vb}) I_{vb}(t) \quad (14)$$

with

$$N_h = S_a(t) + E_a(t) + I_a(t) + R_a(t) + S_b(t) + E_b(t) + I_b(t) + R_b(t), \quad (15)$$

$$N_{va} = S_{va}(t) + E_{va}(t) + I_{va}(t), \quad N_{vb} = S_{vb}(t) + E_{vb}(t) + I_{vb}(t). \quad (16)$$

The parameters of our dynamical system are defined in Table 1. Taking the time derivative of N_h and N_{va} (and N_{vb}) and substituting the time derivatives of the subpopulations equations (1) to (14), we find that

$$\frac{dN_h}{dt} = 0, \quad \frac{dN_{va}}{dt} = 0 \quad \text{and} \quad \frac{dN_{vb}}{dt} = 0$$

meaning that N_h , N_{va} and N_{vb} are independent of time, i.e., they are constants. Dividing equations (1)-(14) by these constants and introducing the new variables,

$$\begin{aligned} \tilde{S}_a(t) &= \frac{S_a(t)}{N_h}, \quad \tilde{E}_a(t) = \frac{E_a(t)}{N_h}, \quad \tilde{I}_a(t) = \frac{I_a(t)}{N_h}, \quad \tilde{R}_a(t) = \frac{R_a(t)}{N_h}, \\ \tilde{S}_b(t) &= \frac{S_b(t)}{N_h}, \quad \tilde{E}_b(t) = \frac{E_b(t)}{N_h}, \quad \tilde{I}_b(t) = \frac{I_b(t)}{N_h}, \\ \tilde{R}_b(t) &= \frac{R_b(t)}{N_h}, \quad \tilde{S}_{va}(t) = \frac{S_{va}(t)}{V_a/(\delta_{va} + C_a)}, \quad \tilde{E}_{va}(t) = \frac{E_{va}(t)}{V_a/(\delta_{va} + C_a)}, \\ \tilde{I}_{va}(t) &= \frac{I_{va}(t)}{V_a/(\delta_{va} + C_a)}, \quad \tilde{S}_{vb}(t) = \frac{S_{vb}(t)}{V_b/(\delta_{vb} + C_b)}, \\ \tilde{E}_{vb}(t) &= \frac{E_{vb}(t)}{V_b/(\delta_{vb} + C_b)}, \quad \tilde{I}_{vb}(t) = \frac{I_{vb}(t)}{V_b/(\delta_{vb} + C_b)}, \end{aligned}$$

we obtain the new set of equations:

$$\frac{d\tilde{S}_a(t)}{dt} = a - \left(T_{va}\Psi_a\gamma_{vah} \frac{\tilde{I}_{va}(t)(V_a/(\delta_{va} + C_a))}{N_h + b} + \delta_h \right) \tilde{S}_a(t) + \theta_a \tilde{R}_a(t), \tag{17}$$

$$\frac{d\tilde{E}_a(t)}{dt} = \left(T_{va}\Psi_a\gamma_{vah} \frac{\tilde{I}_{va}(t)(V_a/(\delta_{va} + C_a))}{N_h + b} \right) \tilde{S}_a(t) - (\delta_h + \alpha_a) \tilde{E}_a(t), \tag{18}$$

$$\frac{d\tilde{I}_a(t)}{dt} = \alpha_a \tilde{E}_a(t) - (\delta_h + \beta_a) \tilde{I}_a(t), \tag{19}$$

$$\frac{d\tilde{S}_b(t)}{dt} = (1 - a) - \left(T_{vb}\Psi_b\gamma_{vbh} \frac{\tilde{I}_{vb}(t)(V_b/(\delta_{vb} + C_b))}{N_h + b} + \delta_h \right) \tilde{S}_b(t) + \theta_b \tilde{R}_b(t), \tag{20}$$

$$\frac{d\tilde{E}_b(t)}{dt} = \left(T_{vb}\Psi_b\gamma_{vbh} \frac{\tilde{I}_{vb}(t)(V_b/(\delta_{vb} + C_b))}{N_h + b} \right) \tilde{S}_b(t) - (\delta_h + \alpha_b) \tilde{E}_b(t), \tag{21}$$

$$\frac{d\tilde{I}_b(t)}{dt} = \alpha_b \tilde{E}_b(t) - (\delta_h + \beta_b) \tilde{I}_b(t), \tag{22}$$

$$\begin{aligned} \frac{d\tilde{E}_{va}(t)}{dt} = & \left(A_a(1 + \varepsilon_a \sin \tau(t + 194)) T_{va} \Psi_a \gamma_{hva} \frac{(P_a + \tilde{I}_a(t) * N_h)}{N_h + b} \right) \\ & \times (1 - \tilde{E}_{va}(t) - \tilde{I}_{va}(t)) - (C_a + \delta_{va} - \beta_{va}) \tilde{E}_{va}(t), \end{aligned} \tag{23}$$

$$\frac{d\tilde{I}_{va}(t)}{dt} = \beta_{va} \tilde{E}_{va}(t) - (C_a + \delta_{va}) \tilde{I}_{va}(t), \tag{24}$$

$$\begin{aligned} \frac{d\tilde{E}_{vb}(t)}{dt} = & \left(A_b(1 + \varepsilon_b \sin \tau(t + 240)) T_{vb} \Psi_b \gamma_{hvb} \frac{(P_a + \tilde{I}_b(t) * N_h)}{N_h + b} \right) \\ & \times (1 - \tilde{E}_{vb}(t) - \tilde{I}_{vb}(t)) - (C_b + \delta_{vb} - \beta_{vb}) \tilde{E}_{vb}(t), \end{aligned} \tag{25}$$

$$\frac{d\tilde{I}_{vb}(t)}{dt} = \beta_{vb} \tilde{E}_{vb}(t) - (C_b + \delta_{vb}) \tilde{I}_{vb}(t) \tag{26}$$

with $\tilde{S}_a + \tilde{E}_a + \tilde{I}_a + \tilde{R}_a = 1$, $\tilde{S}_{va} + \tilde{E}_{va} + \tilde{I}_{va} = 1$, $\tilde{S}_{vb} + \tilde{E}_{vb} + \tilde{I}_{vb} = 1$.
 The peak numbers of mosquito population in a given year are determined by $(A_a(1 + \varepsilon_a \sin \tau(t + 194)))$ and $(A_b(1 + \varepsilon_b \sin \tau(t + 240)))$ [19].

Table 1. Definition of variables and parameters in the model

Parameters	Definition
δ_h	The death rate of host population
T_{va}	The development rate of adult female <i>Aedes aegypti</i> mosquitoes [7, 23]
T_{vb}	The development rate of adult female <i>Aedes albopictus</i> mosquitoes [23]
Ψ_a	The biting rate of <i>Aedes aegypti</i> population
Ψ_a	The biting rate of <i>Aedes albopictus</i> population
γ_{vah}	The transmission probability (from <i>Aedes aegypti</i> to human population)
γ_{vbh}	The transmission probability (from <i>Aedes albopictus</i> to human population)

γ_{hva}	The transmission probability (from human population to <i>Aedes aegypti</i>)
γ_{hvb}	The transmission probability (from human population to <i>Aedes albopictus</i>)
α_a	The viral development rate of <i>Aedes aegypti</i> in human
α_b	The viral development rate of <i>Aedes albopictus</i> in human
β_{va}	The viral development rate of <i>Aedes aegypti</i> bodies
β_{vb}	The viral development rate of <i>Aedes albopictus</i> bodies
β_a	The recovery rate of human population who be infected with <i>Aedes aegypti</i>
β_b	The recovery rate of human population who be infected with <i>Aedes albopictus</i>
δ_{va}	The death rate of <i>Aedes aegypti</i>
δ_{vb}	The death rate of <i>Aedes albopictus</i>
C_a and C_b	The control effort rates in <i>Aedes aegypti</i> and <i>Aedes albopictus</i>
ε_a and ε_b	The measure of influence on the transmission
V_a	The constant recruitment rate of <i>Aedes aegypti</i>
V_b	The constant recruitment rate of <i>Aedes albopictus</i>

In this paper, we took the period of *A. aegypti* reproduction to be 194 and of the *A. albopictus* to be 240.

3. Equilibrium Points for SEIRS Model

The equilibrium points are obtained by setting the RHS of equations (17)-(26) to zero. We only consider solutions in the domains

$$\Omega_a = \{(\tilde{S}_a, \tilde{E}_a, \tilde{I}_a, \tilde{E}_{va}, \tilde{I}_{va}) : \tilde{S}_a, \tilde{E}_a, \tilde{I}_a, \tilde{E}_{va}, \tilde{I}_{va} \geq 0, \\ \tilde{S}_a + \tilde{E}_a + \tilde{I}_a, \tilde{E}_{va} + \tilde{I}_{va} \leq 1\}$$

$$\Omega_b = \{(\tilde{S}_b, \tilde{E}_b, \tilde{I}_b, \tilde{E}_{vb}, \tilde{I}_{vb}) : \tilde{S}_b, \tilde{E}_b, \tilde{I}_b, \tilde{E}_{vb}, \tilde{I}_{vb} \geq 0, \\ \tilde{S}_b + \tilde{E}_b + \tilde{I}_b, \tilde{E}_{vb} + \tilde{I}_{vb} \leq 1\}$$

since the flow generated by vector field of equations (17)-(26) in these regions will be positive invariant and the trajectories must not pass into the exterior region of Ω_a and Ω_b otherwise some of the populations would become negative which is impossible. The equilibrium points in our model are

(i) Disease free equilibrium point: $S_{0a} = (1, 0, 0, 0, 0)$ if the infecting mosquito is *A. aegypti* and $S_{0b} = (1, 0, 0, 0, 0)$ if the infecting mosquito is *A. albopictus*.

(ii) Endemic disease equilibrium point: $S_{0a}^* = (S_a^*, E_a^*, I_a^*, E_{va}^*, I_{va}^*)$ when the infecting mosquito is *Aedes aegypti* and $S_{0b}^* = (S_b^*, E_b^*, I_b^*, E_{vb}^*, I_{vb}^*)$ if the infecting mosquito is *A. albopictus*.

The equilibrium populations are

$$S_a^* = \frac{-a - \theta_a + I_a^* \theta_a + \frac{A_a(I_a^* N_h + P_a) \omega_5 (a + \theta_a - I_a^* \theta_a)}{\omega_3 + A_a(I_a^* N_h + P_a) \omega_4}}{\delta_h + \theta_a + \frac{(I_a^* N_h + P_a) \omega_6}{\omega_7 + A_a(I_a^* N_h + P_a) \omega_8}}, \tag{27}$$

$$E_a^* = \frac{A_a(I_a^* N_h + P_a) \beta_{va} (a + \theta_a - I_a^* \theta_a) \omega_1 \omega_2}{\omega_3 + A_a(I_a^* N_h + P_a) \omega_4}, \tag{28}$$

$$I_a^* = \frac{1}{(2A_b N_h (\alpha_a \beta_{va} \theta_a \omega_1 \omega_2 + (\beta_a + \delta_h) \omega_4)) \times (a A_a N_h \alpha_a \beta_{va} \omega_1 \omega_2 + A_a N_h \alpha_a \theta_a \beta_{va} \omega_1 \omega_2 - A_a P_a \alpha_a \beta_{va} \theta_a \omega_1 \omega_2 - \beta_a \omega_3 + \delta_h \omega_3 A_a P_a \beta_a \omega_4 + A_a P_a \delta_h \omega_4) + \sqrt{4(A_a^2 N_h P_a \alpha_a \beta_{va} (a + \theta_a) \omega_1 \omega_2 (\alpha_a \beta_{va} \theta_a \omega_1 \omega_2 + (\beta_a + \delta_h) \omega_4 + (-A_a \alpha_a \beta_{va} (a N_h + (N_h - P_a) \theta_a) \omega_1 \omega_2 + (\beta_a + \delta_h) \omega_3 + A_a P_a (\beta_a + \delta_h) \omega_4)^2))}}, \tag{29}$$

$$E_{va}^* = \frac{A_a(I_a^* N_h + P_a) \omega_2 (-\omega_7 + A_a(I_a^* N_h + P_a) (\beta_{va} \omega_2 - \omega_8))}{(\omega_9 + A_a(I_a^* N_h + P_a) \omega_2) (\omega_7 + A_a(I_a^* N_h + P_a) \omega_8)}, \tag{30}$$

$$I_{va}^* = \frac{A_a(I_a^*N_h + P_a)\beta_{va}\omega_2}{\omega_7 + A_a(I_a^*N_h + P_a)\omega_8}, \tag{31}$$

$$S_b^* = \frac{-1 + a - \theta_b + I_b^*\theta_b + \frac{A_b(I_b^*N_h + P_b)\beta_{vb}\theta_b(-1 + a + (-1 + I_b^*)\theta_b)\mu_1\mu_2}{\mu_3 + A_b(I_b^*N_h + P_b)\mu_4}}{\delta_h + \theta_b + \frac{A_b(I_b^*N_h + P_b)\mu_{10}}{\mu_6 + A_b(I_b^*N_h + P_b)\mu_7}}, \tag{32}$$

$$E_b^* = \frac{A_b(I_b^*N_h + P_b)\beta_{vb}(-1 + a + (-1 + I_b^*)\theta_b)\mu_1\mu_2}{\mu_3 + A_b(I_b^*N_h + P_b)\mu_4}, \tag{33}$$

$$I_b^* = \frac{1}{(2A_bN_h(\alpha_b\beta_{vb}\theta_b\mu_1\mu_2 + (\beta_b + \delta_h)\mu_4)) \times (A_b\alpha_bN_h\alpha_a\beta_{vb}(P_b\theta_b + N_h(1 - a + \theta_b)) + \mu_1\mu_2 + (\beta_b + \delta_h)\mu_3 + A_bP_b(\beta_b + \delta_h)\mu_4) + \sqrt{((\beta_b + \delta_h)^2\mu_3^2 + 2A_b(\beta_b + \delta_h)\mu_3(\alpha_b\beta_{vb}(N_h - aN_h + N_h\theta_b - P_b\theta_b) \times \mu_1\mu_2 + P_b(\beta_b + \delta_h)\mu_4) + (A_b\alpha_b\beta_{vb}((-1 + a)N_h - (N_h + P_b)\theta_b)\mu_1\mu_2 + A_bP_b(\beta_b + \delta_h)\mu_4)^2)},} \tag{34}$$

$$I_{vb}^* = \frac{A_b(I_b^*N_h + P_b)\beta_{vb}\mu_2}{\mu_6 + A_b(I_b^*N_h + P_b)\mu_7}, \tag{35}$$

$$E_{vb}^* = \frac{A_b(I_b^*N_h + P_b)\mu_2(-\mu_6 + A_b(I_b^*N_h + P_b)\mu_9)}{\mu_8 + A_b(I_b^*N_h + P_b)\mu_2(\mu_6 + A_b(I_b^*N_h + P_b)\mu_7)}, \tag{36}$$

where

$$\omega_1 = \frac{\gamma_{vah}T_{va}V_a\Psi_a}{(N_h + b)(\delta_{va} + C_a)}, \quad \omega_2 = \frac{\gamma_{hva}T_{va}\Psi_a(1 + \epsilon_a \sin \tau(t + 194))}{N_h + b},$$

$$\omega_3 = (\alpha_a + \delta_h)(\delta_{va} + C_a)(C_a + \beta_{va} + \delta_{va})(\delta_h + \theta_a),$$

$$\omega_4 = ((\alpha_a + \delta_h)(C_a - \beta_{va} + \delta_{va})(\delta_h + \theta_a) + \beta_{va}(\alpha_a + \delta_h + \theta_a)\omega_1)\omega_2,$$

$$\omega_5 = \beta_{va}\theta_a\omega_1\omega_2, \quad \omega_6 = A_a\beta_{va}\omega_1\omega_2,$$

$$\omega_7 = (\delta_{va} + C_a)(C_a + \beta_{va} + \delta_{va}), \quad \omega_8 = (C_a - \beta_{va} + \delta_{va})\omega_2,$$

$$\omega_9 = C_a + \beta_{va} + \delta_{va},$$

$$\mu_1 = \frac{\gamma_{vbh}I_{vb}V_b\Psi_b}{(N_h + b)(\delta_{vb} + C_b)}, \quad \mu_2 = \frac{\gamma_{vbh}I_{vb}\Psi_b(1 + \varepsilon_b \sin \tau(t + 240))}{N_h + b},$$

$$\mu_3 = (\alpha_b + \delta_h)(\delta_{vb} + C_b)(C_b + \beta_{vb} + \delta_{vb})(\delta_h + \theta_b),$$

$$\mu_4 = ((\alpha_b + \delta_h)(C_b - \beta_{vb} + \delta_{vb})(\delta_h + \theta_b) + \beta_{vb}(\alpha_b + \delta_h - \theta_b)\mu_1)\mu_2,$$

$$\mu_6 = (\delta_{vb} + C_b)(C_b + \beta_{vb} + \delta_{vb}),$$

$$\mu_7 = (C_b - \beta_{vb} + \delta_{vb})\mu_2, \quad \mu_8 = C_b + \beta_{vb} + \delta_{vb},$$

$$\mu_9 = \beta_{vb}\mu_2 - \mu_7, \quad \mu_{10} = \beta_{vb}\mu_1\mu_2.$$

4. Local Stability of SEIRS Model

The local stability of an equilibrium point is determined from the Jacobian matrix of the system of equations (17)-(26) evaluated at the equilibrium points. If all the eigenvalues have negative real parts, then the equilibrium point will be locally asymptotically stable. The standard dynamical modeling method [20] is used in this study. From (17)-(26), the Jacobian matrices are:

$$A_{aa} =$$

$$\begin{pmatrix} -\left(T_{m\psi_a}\gamma_{am}\frac{I_m(t)N\psi_a(\delta_{am} + C_a)}{N_h + b} + \delta_a\right) & 0 & 0 & 0 & -\left(T_{m\psi_a}\gamma_{am}\frac{I_m(\delta_{am} + C_a)}{N_h + b}\right)\delta_a(t) \\ \left(T_{m\psi_a}\gamma_{am}\frac{I_m(t)N\psi_a(\delta_{am} + C_a)}{N_h + b}\right) & -(\delta_h + \alpha_a) & 0 & 0 & \left(T_{m\psi_a}\gamma_{am}\frac{I_m(\delta_{am} + C_a)}{N_h + b}\right)\delta_a(t) \\ 0 & \alpha_a & -(\delta_h + \beta_a) & 0 & 0 \\ 0 & 0 & \left(A_a(1 + \varepsilon_a \sin \tau(t + 194))T_{m\psi_a}\gamma_{am}\frac{I_m(\delta_{am} + C_a)}{N_h + b}\right) & -(C_a + \delta_{am} - \beta_{am}) & 0 \\ 0 & 0 & \times(1 - E_{m\psi_a}(t) - T_{m\psi_a}(t)) & 0 & 0 \\ 0 & 0 & 0 & \beta_{am} & -(C_a + \delta_{am}) \end{pmatrix}$$

(37a)

$$A_{bb} =$$

$$\begin{pmatrix} -\left(T_{10} \Psi_{10} \gamma_{10} \frac{T_{10}(t)(V_{10}/(\delta_{10} + C_{10}))}{N_A + b} + \delta_{10}\right) & 0 & 0 & 0 & -\left(T_{10} \Psi_{10} \gamma_{10} \frac{(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) \bar{S}_B(t) \\ \left(T_{10} \Psi_{10} \gamma_{10} \frac{T_{10}(t)(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) & -(\delta_{10} + \alpha_{10}) & 0 & 0 & \left(T_{10} \Psi_{10} \gamma_{10} \frac{(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) \bar{S}_B(t) \\ 0 & \alpha_{10} & -(\delta_{10} + \beta_{10}) & 0 & 0 \\ 0 & 0 & \left(A_0(1 + c_{10} \sin(t + 240)) T_{10} \Psi_{10} \gamma_{10} \frac{(B_0 + N_{A1})}{N_A + b}\right) & -(C_{10} + \delta_{10} - \beta_{10}) & 0 \\ & & \times (1 - E_{10}(t) - T_{10}(t)) & & \\ 0 & 0 & 0 & \beta_{10} & -(C_{10} + \delta_{10}) \end{pmatrix} \quad (37b)$$

The characteristic equation is determined from

$$|A_{aa} - \lambda I| =$$

$$\begin{pmatrix} -\left(T_{10} \Psi_{10} \gamma_{10} \frac{T_{10}(t)(V_{10}/(\delta_{10} + C_{10}))}{N_A + b} + \delta_{10}\right) - \lambda & 0 & 0 & 0 & -\left(T_{10} \Psi_{10} \gamma_{10} \frac{(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) \bar{S}_B(t) \\ \left(T_{10} \Psi_{10} \gamma_{10} \frac{T_{10}(t)(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) & -(\delta_{10} + \alpha_{10}) - \lambda & 0 & 0 & \left(T_{10} \Psi_{10} \gamma_{10} \frac{(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) \bar{S}_B(t) \\ 0 & \alpha_{10} & -(\delta_{10} + \beta_{10}) - \lambda & 0 & 0 \\ 0 & 0 & \left(A_0(1 + c_{10} \sin(t + 194)) T_{10} \Psi_{10} \gamma_{10} \frac{(B_0 + N_{A1})}{N_A + b}\right) & -(C_{10} + \delta_{10} - \beta_{10}) - \lambda & 0 \\ & & \times (1 - E_{10}(t) - T_{10}(t)) & & \\ 0 & 0 & 0 & \beta_{10} & -(C_{10} + \delta_{10}) - \lambda \end{pmatrix} = 0 \quad (38a)$$

when A_{aa} is Jacobian matrix for endemic equilibrium point of *Aedes aegypti*, λ is the eigenvalue, I is the identity matrix and

$$|A_{bb} - \lambda I| =$$

$$\begin{pmatrix} -\left(T_{10} \Psi_{10} \gamma_{10} \frac{T_{10}(t)(V_{10}/(\delta_{10} + C_{10}))}{N_A + b} + \delta_{10}\right) - \lambda & 0 & 0 & 0 & -\left(T_{10} \Psi_{10} \gamma_{10} \frac{(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) \bar{S}_B(t) \\ \left(T_{10} \Psi_{10} \gamma_{10} \frac{T_{10}(t)(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) & -(\delta_{10} + \alpha_{10}) - \lambda & 0 & 0 & \left(T_{10} \Psi_{10} \gamma_{10} \frac{(V_{10}/(\delta_{10} + C_{10}))}{N_A + b}\right) \bar{S}_B(t) \\ 0 & \alpha_{10} & -(\delta_{10} + \beta_{10}) - \lambda & 0 & 0 \\ 0 & 0 & \left(A_0(1 + c_{10} \sin(t + 240)) T_{10} \Psi_{10} \gamma_{10} \frac{(B_0 + N_{A1})}{N_A + b}\right) & -(C_{10} + \delta_{10} - \beta_{10}) - \lambda & 0 \\ & & \times (1 - E_{10}(t) - T_{10}(t)) & & \\ 0 & 0 & 0 & \beta_{10} & -(C_{10} + \delta_{10}) - \lambda \end{pmatrix} = 0 \quad (38b)$$

when A_{bb} is the Jacobian matrix for endemic equilibrium point of *A. albopictus* case, λ is the eigenvalue, I is the identity matrix.

The eigenvalues of $|A_{aa} - \lambda I|$ in *A. aegypti* and $|A_{bb} - \lambda I|$ in *A. albopictus* are obtained by solving $\det(A_{aa} - \lambda I_5) = 0$ and $\det(A_{bb} - \lambda I_5) = 0$, where A_{aa} and A_{bb} are Jacobian matrix for the two species, equations (37a) and (37b), respectively. We are interested in the stabilities of the endemic equilibrium points.

Lemma. *If $S_0 > 1$, then the endemic equilibrium points S_{0a}^* and S_{0b}^* of equations (17)-(26) are locally asymptotically stable in Ω_a and Ω_b , with the basic reproduction number S_0 is given by [22],*

$$S_0 = \text{Max}\{S_{0a}, S_{0b}\},$$

where

$$S_{0a} = (aA_a N_h \alpha_a \beta_{va} \omega_1 \omega_2 + A_a N_h \alpha_a \beta_{va} \theta_a \omega_1 \omega_2 + \delta_h \omega_3 + \sqrt{4(A_a^2 N_h P_a \alpha_a \beta_{va} (a + \theta_a) \omega_1 \omega_2 (\alpha_a \beta_{va} \theta_a \omega_1 \omega_2 + (\beta_a + \delta_h) \omega_4 + (-A_a \alpha_a \beta_{va} (aN_h + (N_h - P_a) \theta_a) \omega_1 \omega_2 + (\beta_a + \delta_h) \omega_3 + A_a P_a (\beta_a + \delta_h) \omega_4)^2))}) / (\beta_a \omega_3 + A_b P_a (\alpha_a \theta_a \beta_{va} \omega_1 \omega_2 + (\beta_a + \delta_h) \omega_4)), \tag{39}$$

$$S_{0b} = (A_b N_h \alpha_b \beta_{vb} \mu_1 \mu_2 + A_b N_h \alpha_b \beta_{vb} \theta_b \mu_1 \mu_2 + \beta_b \mu_3 + \delta_h \mu_3 + A_b P_b \beta_b \mu_4 + A_b P_b \delta_h \mu_4 + \sqrt{((\beta_b + \delta_h)^2 \mu_3^2 + 2A_b (\beta_b + \delta_h) \mu_3 (\alpha_b \beta_{vb} (N_h - aN_h + N_h \theta_b - P_b \theta_b) \mu_1 \mu_2 + P_b (\beta_b + \delta_h) \mu_4) + (A_b \alpha_b \beta_{vb} ((-1 + a) N_h - (N_h + P_b) \theta_b) \mu_1 \mu_2 + A_b P_b (\beta_b + \delta_h) \mu_4)^2)}) / a(A_b N_h \alpha_b \beta_{vb} \mu_1 \mu_2 + A_b \alpha_b P_b \beta_{vb} \theta_b \mu_1 \mu_2) \tag{40}$$

with

$$\Omega_a = \{(\tilde{S}_a, \tilde{E}_a, \tilde{I}_a, \tilde{E}_{va}, \tilde{I}_{va}) : \tilde{S}_a, \tilde{E}_a, \tilde{I}_a, \tilde{E}_{va}, \tilde{I}_{va} \geq 0, \tilde{S}_a + \tilde{E}_a + \tilde{I}_a, \tilde{E}_{va} + \tilde{I}_{va} \leq 1\}.$$

Proof. After evaluating the eigenvalue equation, equation (38a), i.e., we need to solve $\det(A_a - \lambda I_5) = 0$. Doing this, we obtain

$$(\alpha_a + \delta_h + \lambda)(\beta_a + \delta_h + \lambda)(\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3) = 0, \quad (41)$$

where

$$\begin{aligned} A_1 &= 2C_a + \beta_{va} + \delta_h + 2\delta_{va} + \frac{A_a(I_a''N_h + P_a)\beta_{va}\omega_1\omega_2}{\omega_7 + A_a(I_a''N_h + P_a)\omega_8}, \\ A_2 &= C_a^2 + \delta_{va}(2\delta_h + \delta_{va}) \\ &\quad + C_a(\beta_{va} + 2\delta_h + 2\delta_{va} + \frac{2A_a(I_a''N_h + P_a)\beta_{va}\omega_1\omega_2}{\omega_7 + A_a(I_a''N_h + P_a)\omega_8}) \\ &\quad + \beta_{va}(\delta_h + \frac{\delta_{va}(\omega_7 + A_a(I_a''N_h + P_a)(2\omega_1\omega_2 + \omega_8))}{\omega_7 + A_a(I_a''N_h + P_a)\omega_8}) \\ &\quad + (A_aN_h\alpha_a(a + \theta_a - I_a''\theta_a)\omega_1\omega_2(\omega_3 + A_a(I_a''N_h + P_a)(\omega_4 - \omega_5)) \\ &\quad \times (\omega_7 + A_a(I_a''N_h + P_a)\omega_8)(2A_a(I_a''N_h + P_a)\omega_2 + \omega_9))/ \\ &\quad ((\omega_3 + A_a(I_a''N_h + P_a)\omega_4)((\delta_h + \theta_a)\omega_7 + (I_a''N_h + P_a) \\ &\quad \times (\omega_6 + A_a(\delta_h + \theta_a)\omega_8))(A_a(I_a''N_h + P_a)\omega_2 + \omega_9))) \\ &\quad + (A_a(I_a''N_h + P_a)\beta_{va}^2\omega_1\omega_2(A_a(I_a''N_h + P_a)\omega_2\omega_3(I_a''N_h\omega_6 \\ &\quad + P_a\omega_6 + (\delta_h + \theta_a)\omega_7 - 2A_a^4N_h(I_a''N_h + P_a)^3\alpha_a(a + \theta_a - I_a''\theta_a)\omega_2^2 \\ &\quad \times (\omega_4 - \omega_5)\omega_8 + \omega_3(I_a''N_h\omega_6 + P_a^2\omega_4\omega_6 + (\delta_h + \theta_a)\omega_7)\omega_9 \\ &\quad + A_a(I_a''^2N_h^2\omega_4\omega_6 + P_a^2\omega_4\omega_6 - N_h\alpha_a(a + \theta_a)\omega_2\omega_3\omega_7 \\ &\quad + P_a(\delta_h + \theta_a)(\omega_4\omega_7 + \omega_3\omega_8) + I_a''N_h(2P_a\omega_4\omega_6 + \alpha_a\theta_a\omega_2\omega_3\omega_7 \\ &\quad + (\delta_h + \theta_a)(\omega_4\omega_7 + \omega_3\omega_8)))\omega_9 + A_a^2(I_a''N_h + P_a) \\ &\quad \times (I_a''^2N_h^2\omega_2\omega_4\omega_6 + P_a^2\omega_2\omega_4\omega_6 + P_a(\delta_h + \theta_a)(\omega_2\omega_4\omega_7 \\ &\quad + \omega_2\omega_3\omega_8 - \omega_4\omega_8\omega_9) - N_h\alpha_a(a + \theta_a)\omega_2(2\omega_2\omega_3\omega_7 + (\omega_4\omega_7 \end{aligned}$$

$$\begin{aligned}
& -\omega_5\omega_7 + \omega_3\omega_8)\omega_9) + I_a''N(2P_a\omega_2\omega_4\omega_6 + \alpha_a\theta_a\omega_2(2\omega_2\omega_3\omega_7 \\
& + \omega_4\omega_7\omega_9 - \omega_5\omega_7\omega_9 + \omega_3\omega_8\omega_9) + (\delta_h + \theta_a)(\omega_2\omega_4\omega_7 \\
& + \omega_2\omega_3\omega_8 + \omega_4\omega_8\omega_9))) + A_a^2(I_a''N_h + P_a)^2\omega_2(P_a(\delta_h + \theta_a)\omega_4\omega_8 \\
& + aN_h\alpha_a(-2\omega_2(\omega_4\omega_7 - \omega_5\omega_7 + \omega_3\omega_8) + (-\omega_4 + \omega_5)\omega_8\omega_9) \\
& + N_h(I_a''(\delta_h + \theta_a)\omega_4\omega_8 + (-1 + I_a'')\alpha_a\theta_a(2\omega_2(\omega_4\omega_7 - \omega_5\omega_7 \\
& + \omega_3\omega_8) + (-\omega_4 + \omega_5)\omega_8\omega_9))))/((\omega_3 + A_a(I_a''N_h + P_a)\omega_4(\omega_7 \\
& + (I_a''N_h + P_a)\omega_8))((\delta_h + \theta_a)\omega_7 + (I_a''N_h + P_a)(\omega_6 \\
& + (I_a''N_h + P_a)\omega_8))(A_a(I_a''N_h + P_a)\omega_2 + \omega_9)).
\end{aligned}$$

The first two eigenvalues are $\lambda_1 = -\alpha_a - \delta_h$ and $\lambda_2 = -\beta_a - \delta_h$. They are always negative. The other eigenvalues λ_3, λ_4 and λ_5 which are the solutions to the cubic equation

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0. \quad (42)$$

According to the Routh-Hurwitz theory, the three solutions to the characteristic equations in the form of the cubic equations above will all be negative if the coefficients A_1, A_2 and A_3 satisfy the relations

$$A_1 > 0, \quad A_3 > 0, \quad A_1A_2 > 0. \quad (43)$$

We have plotted the values of A_1, A_2 and A_3 for a set of fixed values appropriate to the *A. aegypti* mosquitoes and a range of viral development rates in this specie of mosquitoes β_{va} . It should be remembered that this rate depends on the temperature of the environment. The fixed values of the other parameters are

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.077, \quad N_h = 200,000,$$

$$\beta_a = 1/14, \quad \alpha_a = 1/6, \quad \beta_{va} = 0.11, \quad \psi_a = 1/11, \quad \gamma_{vah} = 0.0048,$$

$$\gamma_{hva} = 0.0086, T_{va} = 0.2, A_a = 3000, P_a = 0.07, C_a = 0.008,$$

$$\theta_a = 1/6, b = 6,000, V_a = 90,000 \text{ and } \delta_{va} = 1/3.$$

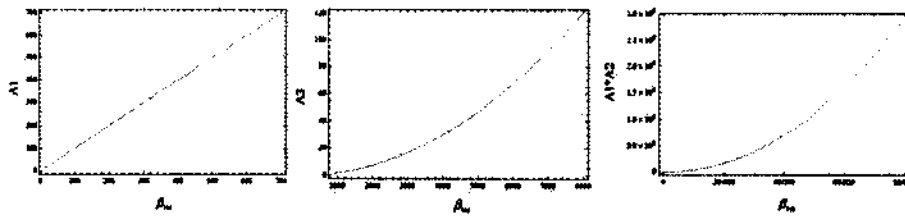


Figure 3. The parameters spaces for the endemic equilibrium point which satisfies the Routh-Hurwitz criteria with the value of parameters in $S_a^*, E_a^*, I_a^*, E_{va}^*$ and I_{va}^* for the parameter values listed in the text.

These values will lead to $S_0 > 1$. Looking at the three graphs in Figure 3, we see that the conditions stated in equation (40) are satisfied and so the equilibrium state S_{0a}^* is locally asymptotically stable.

To see whether S_{0b}^* is locally asymptotically stable, we evaluate equation (38b) to get

$$(\alpha_b + \delta_h + \lambda)(\beta_b + \delta_h + \lambda)(\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3) = 0, \quad (44)$$

where

$$B_1 = 2C_b + \beta_{vb} + \delta_h + 2\delta_{vb} + \frac{A_b(I_b^*N_h + P_b)\beta_{vb}\mu_1\mu_2}{\mu_6 + A_b(I_b^*N_h + P_b)\mu_7},$$

$$B_2 = C_b^2 + \delta_{vb}(2\delta_h + \delta_{vb}) + C_b(\beta_{vb} + 2\delta_h + 2\delta_{vb} + \frac{2A_b(I_b^*N_h + P_b)\beta_{vb}\mu_1\mu_2}{\mu_6 + A_b(I_b^*N_h + P_b)\mu_7}) + (A_b^3N_h(I_b^*N_h + P_b)^2\alpha_b\beta_{vb}^3\theta_b(-1 + a(-1 + I_b^*)\theta_b\mu_1^2\mu_2^3)/((\mu_3 + A_b(I_b^*N_h + P_b)\mu_4)((2\delta_h + \theta_b)\mu_6 + A_b(I_b^*N_h + P_b)(\mu_{10} + (\delta_h + \theta_b)\mu_7))))$$

$$\begin{aligned}
& + \beta_{vb}(\delta_h + \frac{\delta_{vb}(\mu_6 + A_b(I_b^* N_h + P_b)(2\mu_1\mu_2 + \mu_7))}{\mu_6 + A_b(I_b^* N_h + P_b)\mu_7}) \\
& - (A_b N_h \alpha_b (-1 + a(-1 + I_b^*) \theta_b) \mu_1 \mu_2 (\mu_6 \mu_8 \\
& + A_b (I_b^* N_h + P_b) (2\mu_2 \mu_6 + \mu_7 \mu_8) \\
& + A_b^2 (I_b^* N_h + P_b)^2 \mu_2 (\mu_7 - \mu_9))) / (((\delta_h + \theta_b) \mu_6 \\
& + A_b (I_b^* N_h + P_b) (\mu_{10} + (\delta_h + \theta_b) \mu_7)) (A_b (I_b^* N_h + P_b) \mu_2 + \mu_8))) \\
& + (A_b (I_b^* N_h + P_b) \beta_{vb}^2 \mu_1 \mu_2 (A_b (I_b^* N_h + P_b) (\delta_h + \theta_b) \mu_2 \mu_3 \mu_6 \\
& + (\delta_h + \theta_b) \mu_3 \mu_6 \mu_8 + A_b (N_h \alpha_b (-1 + a - \theta_b) \mu_2 \mu_6 (\mu_3 - \theta_b \mu_1 \mu_6) \\
& + P_b (\mu_{10} \mu_3 + (\delta_h + \theta_b) (\mu_4 \mu_6 + \mu_3 \mu_7)) + I_b^* N_h (\mu_{10} \mu_3 \\
& + \alpha_b \theta_b \mu_2 \mu_6 (\mu_3 - \theta_b \mu_1 \mu_6) + (\delta_h + \theta_b) (\mu_4 \mu_6 + \mu_3 \mu_7))) \mu_8 \\
& + A_b^2 (I_b^* N_h + P_b) (P_b \mu_2 (\mu_{10} \mu_3 + (\delta_h + \theta_b) (\mu_4 \mu_6 + \mu_3 \mu_7)) \\
& + P_b \mu_4 (\mu_{10} + (\delta_h + \theta_b) \mu_7) \mu_8 \\
& + N_h \alpha_b (-1 + a - \theta_b) \mu_2 (\mu_2 \mu_6 (\mu_3 - 2\theta_b \mu_1 \mu_6) \\
& + (\mu_4 \mu_6 + (\mu_3 - 2\theta_b \mu_1 \mu_6) \mu_7) \mu_8 + I_b^* N_h (\mu_{10} (\mu_2 \mu_3 + \mu_4 \mu_8) \\
& + (\delta_h + \theta_b) (\mu_2 \mu_4 \mu_6 + \mu_2 \mu_3 \mu_7 + \mu_4 \mu_7 \mu_8) \\
& + \alpha_b \theta_b \mu_2 (\mu_2 \mu_6 (\mu_3 - 2\theta_b \mu_1 \mu_6) + (\mu_4 \mu_6 + \mu_3 \mu_7 - 2\theta_b \mu_1 \mu_6 \mu_7) \mu_8))) \\
& + A_b^4 N_h (I_b^* N_h + P_b)^3 \alpha_b (-1 + a + (-1 + I_b^*) \theta_b) \mu_2^2 \mu_7 (\mu_4 \\
& + \theta_b \mu_1 (-\mu_7 + \mu_9)) + A_b^3 (I_b^* N_h + P_b)^2 \mu_2 (P_b \mu_4 (\mu_{10} \\
& + (\delta_h + \theta_b) \mu_7) + N_h \alpha_b (-1 + a - \theta_b) (\mu_7 (\mu_4 - \theta_b \mu_1 \mu_7) \mu_8 \\
& + \mu_2 (\mu_3 \mu_7 + \mu_6 (\mu_4 \mu_2 + \theta_b \mu_1 (-3\mu_7 + \mu_9)))) \\
& + I_b^* N_h (\mu_{10} \mu_4 + (\delta_h + \theta_b) \mu_4 \mu_7 + \alpha_b \theta_b (\mu_7 (\mu_4 - \theta_b \mu_1 \mu_7) \mu_8
\end{aligned}$$

$$\begin{aligned}
& + \mu_2(\mu_3\mu_7 + \mu_6(\mu_4\mu_2 + \theta_b\mu_1(-3\mu_7 + \mu_9)))))))/ \\
& ((\mu_3 + A_b(I_b^*N_h + P_b)\mu_4)(\mu_6 + A_b(I_b^*N_h + P_b)\mu_7)((\delta_h + \theta_b)\mu_6 \\
& + A_b(I_b^*N_h + P_b)(\mu_{10} + (\delta_h + \theta_b)\mu_7))A_b(I_b^*N_h + P_b)\mu_2 + \mu_8)), \\
B_3 = & \delta_h\delta_{vb}^2 + (C_b^2(\delta_h\mu_6 + A_b(I_b^*N_h + P_b)\beta_{vb}\mu_1\mu_2 + \delta_h\mu_7)))/ \\
& (\mu_6 + A_b(I_b^*N_h + P_b)\mu_7) + (C_b(\beta_{vb} + 2\delta_{vb})\delta_h\mu_6 \\
& + A_b(I_b^*N_h + P_b)(\beta_{vb}\mu_1\mu_2 + \delta_h\mu_7)))/(\mu_6 + A_b(I_b^*N_h + P_b)\mu_7) \\
& + (A_b^3N_h(I_b^*N_h + P_b)^2\alpha_b\beta_{vb}^3\delta_h\theta_b(-1 + a(-1 + I_b^*)\theta_b\mu_1^2\mu_2^3)/ \\
& ((\mu_3 + A_b(I_b^*N_h + P_b)\mu_4)((\delta_h + \theta_b)\mu_6 \\
& + A_b(I_b^*N_h + P_b)(\mu_{10} + (\delta_h + \theta_b)\mu_7))) \\
& + \beta_{vb}(\frac{A_b(I_b^*N_h + P_b)\delta_{vb}^2\mu_1\mu_2}{\mu_6 + A_b(I_b^*N_h + P_b)\mu_7} + (\delta_h(A_b(I_b^*N_h + P_b)\delta_{vb} \\
& \times (\delta_h + \theta_b)\mu_2\mu_6 + \delta_{vb}(\delta_h + \theta_b)\mu_6\mu_8 \\
& + A_b(N_h\alpha_b(1 - a + \theta_b)\mu_1\mu_2\mu_6 + P_b\delta_{vb}(\mu_{10} + (\delta_h + \theta_b)\mu_7) \\
& + I_b^*N_h(-\alpha_b\theta_b\mu_1\mu_2\mu_6 + \delta_{vb}(\mu_{10} + (\delta_h + \theta_b)\mu_7))) \\
& + \mu_8 + A_b^2(I_b^*N_h + P_b)\mu_2(P_b\delta_{vb}(\mu_{10} + (\delta_h + \theta_b)\mu_7) \\
& - N_h\alpha_b(-1 + a - \theta_b)\mu_1(2\mu_2\mu_6 + \mu_7\mu_8) \\
& + I_b^*N_h(\delta_{vb}(\mu_{10} + (\delta_h + \theta_b)\mu_7) - \alpha_b\theta_b\mu_1(2\mu_2\mu_6 + \mu_7\mu_8))) \\
& - A_b^3N_h(I_b^*N_h + P_b)^2\alpha_b(-1 + a(-1 + I_b^*)\theta_b\mu_1\mu_2^2(\mu_7 - \mu_9)))/ \\
& (((\delta_h + \theta_b)\mu_6) + A_b(I_b^*N_h + P_b)((\mu_{10} + (\delta_h + \theta_b)\mu_7)) \\
& \times (A_b(I_b^*N_h + P_b)\mu_2 + \mu_8))) + (A_b(I_b^*N_h + P_b)\beta_{vb}^2\mu_1\mu_2 \\
& \times (A_b(I_b^*N_h + P_b)\delta_{vb}(\delta_h + \theta_b)\mu_2\mu_3\mu_6 + \delta_{vb}(\delta_h + \theta_b)\mu_3\mu_6\mu_8
\end{aligned}$$

$$\begin{aligned}
& + A_b(N_h\alpha_b\delta_h(-1+a-\theta_b)\mu_2\mu_6(\mu_3-\theta_b\mu_1\mu_6) \\
& + P_b\delta_{vb}(\mu_{10}\mu_3+(\delta_h+\theta_b)(\mu_4\mu_6+\mu_3\mu_7)) \\
& + I_b^*N_h(\alpha_b\delta_h\theta_b\mu_2\mu_6(\mu_3-\theta_b\mu_1\mu_6)+\delta_{vb}(\mu_{10}\mu_3 \\
& +(\delta_h+\theta_b)(\mu_4\mu_6+\mu_3\mu_7))))\mu_8+A_b^2(I_b^*N_h+P_b) \\
& \times(N_h\alpha_b\delta_h(-1+a-\theta_b)\mu_2(\mu_2\mu_6(\mu_3-2\theta_b\mu_1\mu_6) \\
& +(\mu_4\mu_6+\mu_3\mu_7-2\theta_b\mu_1\mu_6\mu_7)\mu_8)+P_b\delta_{vb}(\mu_{10}(\mu_2\mu_3+\mu_4\mu_8) \\
& +(\delta_h+\theta_b)(\mu_2\mu_4\mu_6+\mu_2\mu_3\mu_7+\mu_4\mu_7\mu_8)) \\
& +\delta_{vb}\mu_4(\mu_{10}+(\delta_h+\theta_b)\mu_7)\mu_8+\alpha_b\delta_h\theta_b\mu_2(\mu_2\mu_6(\mu_3-2\theta_b\mu_1\mu_6) \\
& +(\mu_4\mu_6+\mu_3\mu_7-2\theta_b\mu_1\mu_6\mu_7)\mu_8))+A_b^4N_h(I_b^*N_h+P_b)^3 \\
& \times\alpha_b\delta_h(-1+a+(-1+I_b^*)\theta_b)\mu_2^2\mu_7(\mu_4+\theta_b\mu_1(-\mu_7+\mu_9)) \\
& +A_b^3(I_b^*N_h+P_b)^2\mu_2(P_b\delta_{vb}\mu_4(\mu_{10}+(\delta_h+\theta_b)\mu_7) \\
& +N_h\alpha_b\delta_h(-1+a-\theta_b)(\mu_7(\mu_4-\theta_b\mu_1\mu_7)\mu_8+\mu_2(\mu_3\mu_7 \\
& +\mu_6(\mu_4+\theta_b\mu_1(-3\mu_7+\mu_9)))))+I_b^*N_h(\delta_{vb}\mu_4(\mu_{10}+(\delta_h+\theta_b)\mu_7) \\
& +\alpha_b\delta_h\theta_b(\mu_7(\mu_4-\theta_b\mu_1\mu_7)\mu_8+\mu_2(\mu_3\mu_7+\mu_6(\mu_4 \\
& +\theta_b\mu_1(-3\mu_7+\mu_9)))))))/((\mu_3+A_b(I_b^*N_h+P_b)\mu_4) \\
& \times(\mu_6+A_b(I_b^*N_h+P_b)\mu_7)((\delta_h+\theta_b)\mu_6+A_b(I_b^*N_h+P_b) \\
& \times(\mu_{10}+(\delta_h+\theta_b)\mu_7))A_b(I_b^*N_h+P_b)\mu_2+\mu_8)).
\end{aligned}$$

Again we see that the first two eigenvalues $\lambda_1 = -\alpha_b - \delta_h$ and $\lambda_2 = -\beta_b - \delta_h$ are negative. The other three eigenvalues are the solutions of the characteristic equation $(\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3) = 0$, where the B 's are given above. Just like before, the three eigenvalues will be negative if (i) $B_1 > 0$,

(ii) $B_3 > 0$, (iii) $B_1 B_2 > 0$. As before, we have evaluated the values of the B 's numerically using the following parameter values:

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.077, \quad N_h = 200,000,$$

$$\beta_b = 1/19, \quad \alpha_b = 1/13, \quad \beta_{vb} = 0.8, \quad \psi_b = 1/19, \quad \gamma_{vbh} = 0.083,$$

$$\gamma_{hvb} = 0.068, \quad T_{vb} = 0.6, \quad A_b = 35000, \quad P_b = 0.97,$$

$$C_b = 0.7, \quad \theta_b = 1/9, \quad b = 10,000, \quad V_b = 960,000 \text{ and } \delta_{va} = 1/9.$$

These values are appropriate for the *A. albopictus* mosquitoes. Looking at Figure 4, we see that for the values of the parameters used, the B 's satisfy the Routh-Hurwitz criteria and so will be locally asymptotically stable. The expressions for S_b^* , E_b^* , I_b^* , E_{vb}^* and I_{vb}^* are defined by equations (32)-(36).

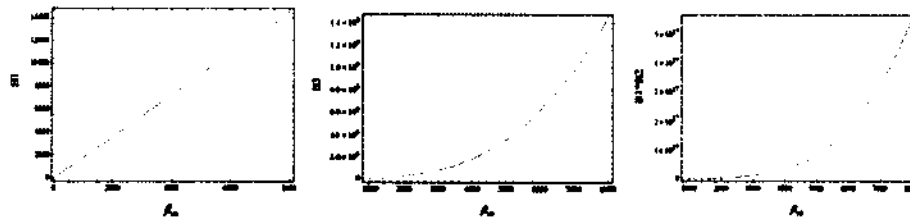


Figure 4. The parameters spaces for the endemic equilibrium point which satisfies the Routh-Hurwitz criteria with the value of parameters listed in the text.

5. Numerical Simulation

After showing that the equilibrium populations during a dengue epidemic in which two species of mosquitoes, *A. aegypti* and *A. albopictus* are co-circulating is locally asymptotically stable, we now simulate the trajectories of the epidemic by numerically solving equations (17) to (26) [23, 24]. The values of parameters used in the numerical simulations are $\delta_h = 1/(365 * 74.6)$ per day, corresponding to a life expectancy of 74.6 years; $\alpha_a = 1/6$ and $\alpha_b = 1/17$, which corresponds to the exposed rate of human

population; $\beta_a = 1/6$ and $\beta_b = 1/19$, corresponding to the recovery rate of human population due to biting of *A. aegypti* and *A. albopictus*, respectively. The transmission probability of *A. aegypti* (ψ_a) and *A. albopictus* (ψ_b) are arbitrary chosen. We assume that no alternative host. The other parameters were arbitrarily chosen. The numerical solutions of (16)-(26) are shown in following figures.

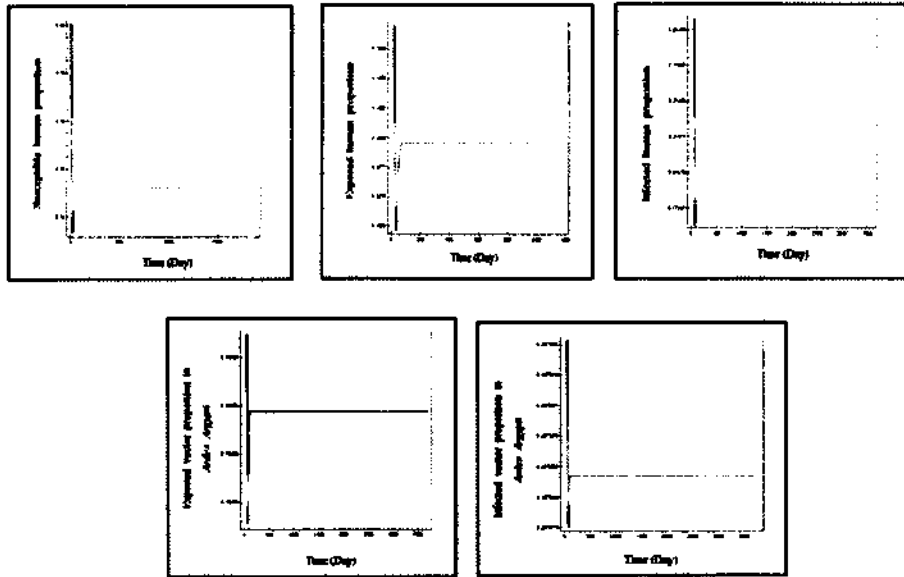


Figure 5. Numerical solutions of (16)-(26), which shows the times series behavior of populations (when $S_0 > 1$) of S_a'' , E_a'' , I_a'' , E_{va}'' and I_{va}'' .

For the case of *Aedes Aegypti* when seasonal change in the total number of this species does not influence the behavior (achieved by setting $\epsilon_a = 0$) of the populations. To obtain the trajectories, we have the following numerical values:

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.00077, \quad N_h = 200,000,$$

$$\beta_a = 1/14, \quad \alpha_a = 1/9, \quad \beta_{va} = 0.11, \quad \psi_a = 1/16, \quad \gamma_{vah} = 0.00018,$$

$$\gamma_{hva} = 0.66, \quad T_{va} = 0.05, \quad A_a = 4000, \quad P_a = 0.09, \quad C_a = 0.008,$$

$$\theta_a = 1/6, \quad b = 3,000, \quad V_a = 9,000,000, \quad \varepsilon_a = 0 \text{ and } \delta_{va} = 1/3,$$

$$S_{0a}^* = 31.0712.$$

These values lead to the following equilibrium states: (0.5323, 0.1079, 0.2516, 0.5820, 0.2872).

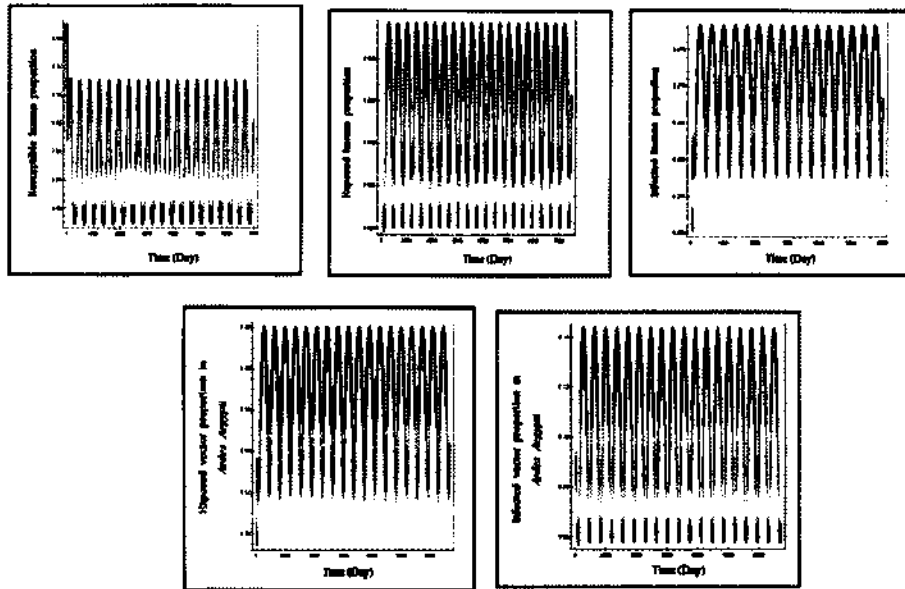


Figure 6. Numerical solutions demonstrate the solution trajectories of S_a'' , E_a'' , I_a'' , E_{va}'' and I_{va}'' . The values of the parameters are given in the text of the paper.

Under the seasonal variation ($\varepsilon_a \neq 0$), the numerical values of the parameters are changed to

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.000007777, \quad N_h = 200,000,$$

$$\beta_a = 1/14, \quad \alpha_a = 1/6, \quad \beta_{va} = 0.11, \quad \psi_a = 1/18, \quad \gamma_{vah} = 0.00018,$$

$$\gamma_{hva} = 0.0026, \quad T_{va} = 0.4, \quad A_a = 7000, \quad P_a = 0.7, \quad C_a = 0.08,$$

$$\theta_a = 1/6, \quad b = 6,000, \quad V_a = 90,000,000, \quad \varepsilon_a = 0.5 \text{ and } \delta_{va} = 1/3.$$

Plotting the series development of two of the populations on the same figures, we see limit cycle behaviors. The values of the parameters are the same as those used in Figure 6.

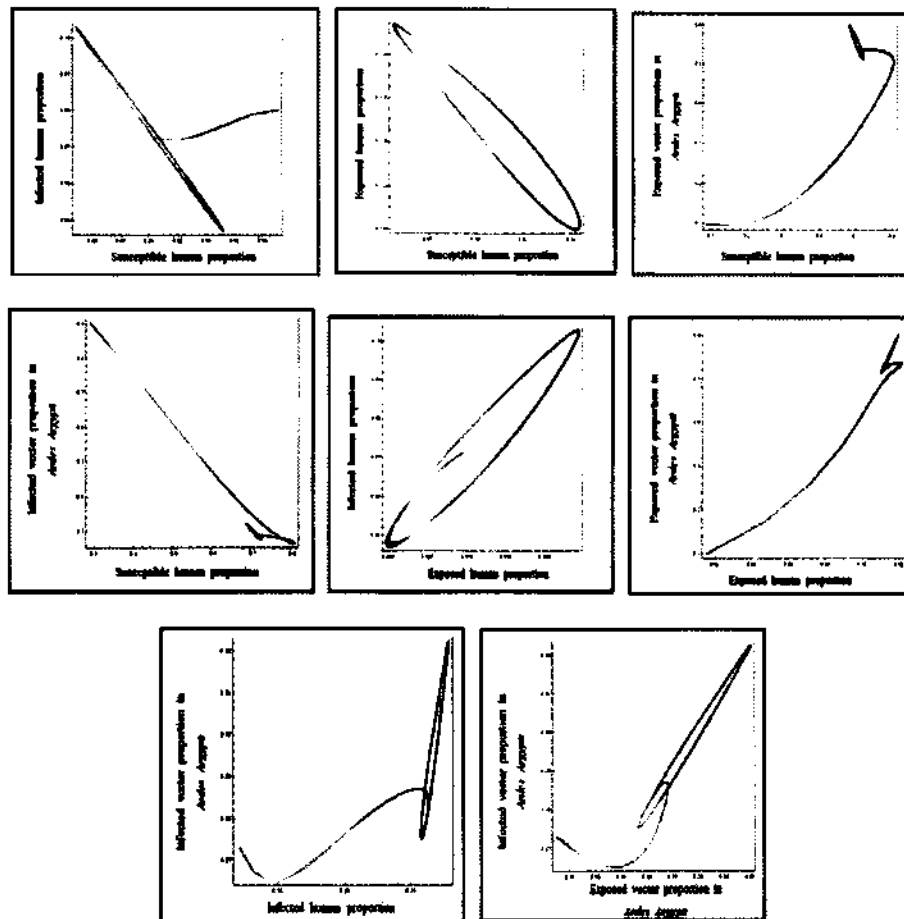


Figure 7. Numerical solutions demonstrate the solution trajectories, projected onto the (S_a^n, I_a^n) , (S_a^n, E_a^n) , (S_a^n, E_{va}^n) , (S_a^n, I_{va}^n) , (E_a^n, I_a^n) , (E_a^n, E_{va}^n) , (I_a^n, I_{va}^n) and (E_{va}^n, I_{va}^n) space.

In case when the infecting mosquitoes are *A. Albopictus* when the seasonal changes of the mosquitoes population is negligible (achieved by setting $\epsilon_b = 0$), the numerical values of the parameters used in solving equations are now changed to:

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.00077, \quad N_h = 200,000,$$

$$\beta_b = 1/19, \quad \alpha_b = 1/18, \quad \beta_{vb} = 0.8, \quad \psi_b = 1/12, \quad \gamma_{vbh} = 0.093,$$

$$\gamma_{hvb} = 0.48, \quad T_{vb} = 0.8, \quad A_b = 45000, \quad P_b = 0.00697, \quad C_b = 0.00007,$$

$$\theta_b = 1/9, \quad b = 100, \quad V_b = 660,000, \quad \epsilon_b = 0 \text{ and } \delta_{vb} = 1/3.$$

These values lead to equilibrium state population of $S_{0b}^* = 1.8572$. The time evolutions of the different population are plotted in Figure 8. This figure corresponds to Figure 5 for the time evolutions of the different populations when the infecting mosquito is the *A. aegypti* mosquito. The endemic equilibrium points are: (0.6305, 0.9272, 0.9781, 0.2000, 0.7998).

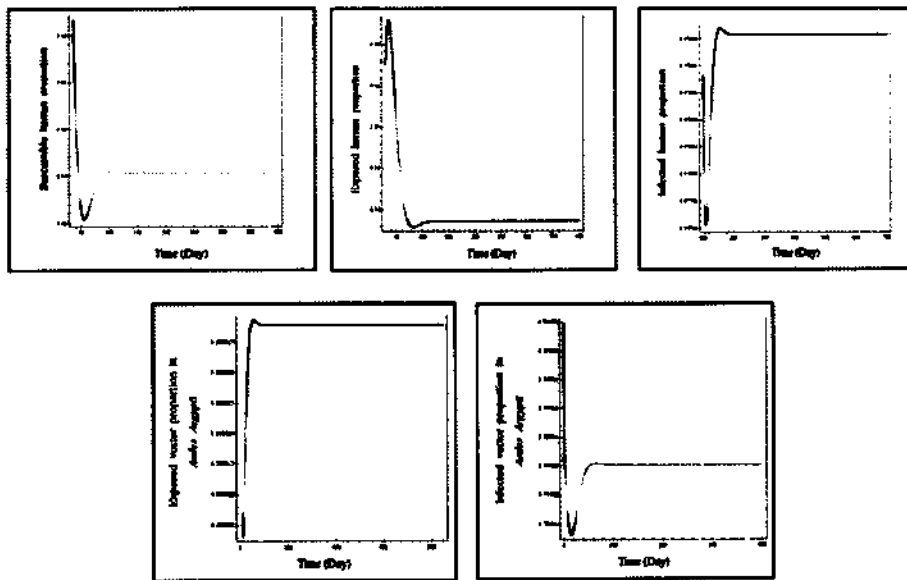


Figure 8. Numerical solutions of (16)-(26) show the time evolutions of S_b^* , E_b^* , I_b^* , E_{vb}^* and I_{vb}^* when the mosquitoes responsible for the epidemic is *A. albopictus*. The values of the parameters are given in the text.

In case when the influence of the seasonal variation of the number of *A. albopictus* is taken into account (achieved by setting $\epsilon_b \neq 0$), the values of

the parameters are now set to

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.000007777, \quad N_h = 200,000,$$

$$\beta_b = 1/19, \quad \alpha_b = 1/12, \quad \beta_{vb} = 0.8, \quad \psi_b = 1/7, \quad \gamma_{vbh} = 0.073,$$

$$\gamma_{hvb} = 0.068, \quad T_{vb} = 0.04, \quad A_b = 25000, \quad P_b = 0.03, \quad C_b = 0.08,$$

$$\theta_b = 1/9, \quad b = 20,000, \quad V_b = 3,000,000, \quad \varepsilon_b = 0.6 \text{ and } \delta_{vb} = 1/5.$$

The time series behaviors of the different populations are shown on Figure 9.

The values of the parameters used to obtain the curves shown in Figure 9 are

$$\theta = \frac{2\pi}{365}, \quad \delta_h = 1/(365 * 74.6)\text{day}^{-1}, \quad a = 0.000007777, \quad N_h = 200,000,$$

$$\beta_b = 1/19, \quad \alpha_b = 1/12, \quad \beta_{vb} = 0.8, \quad \psi_b = 1/7, \quad \gamma_{vbh} = 0.073,$$

$$\gamma_{hvb} = 0.068, \quad T_{vb} = 0.04, \quad A_b = 25000, \quad P_b = 0.03, \quad C_b = 0.08,$$

$$\theta_b = 1/9, \quad b = 20,000, \quad V_b = 3,000,000, \quad \varepsilon_b = 0.6 \text{ and } \delta_{vb} = 1/5.$$

Pairing up the solutions shown in Figure 9 and plotting them on a two plot i.e., (S_b^*, E_b^*) , (S_b^*, E_{vb}^*) , (S_b^*, I_b^*) , (S_b^*, I_{vb}^*) , (I_b^*, E_b^*) , (I_b^*, E_{vb}^*) , (I_b^*, I_{vb}^*) , (E_b^*, E_{vb}^*) and (E_b^*, I_{vb}^*) we see the limit cycle behaviors appearing on Figure 10. Similar limit cycle behaviors to those seen in Figures 7 and 10 will be seen when ε_a is between 0.25 and 0.7 and when ε_b is between 0.3 and 0.65. There appears to be a loss of stability when other values are changed especially changes in the biting rates of the two species, ψ_a (biting rate of *A. aegypti* mosquitoes) and ψ_b (biting rate of *A. albopictus* mosquitoes).

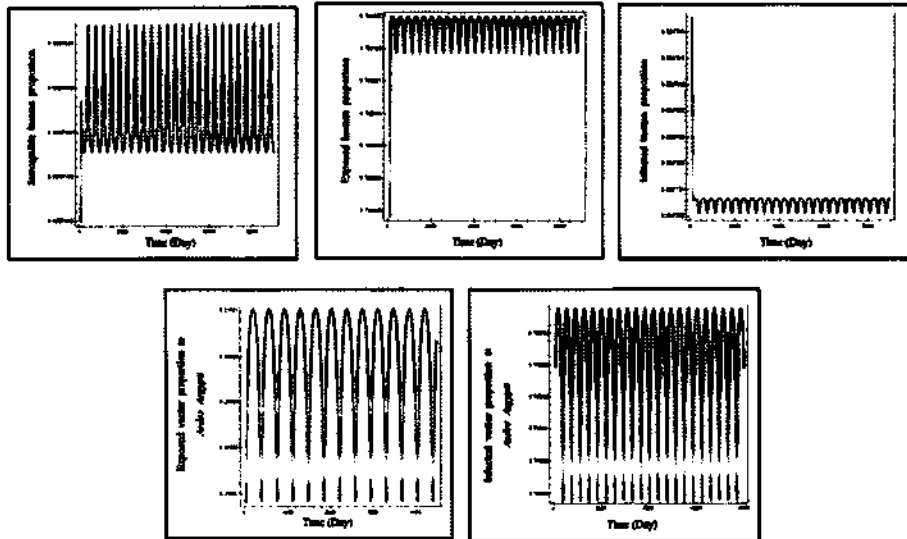


Figure 9. Numerical solutions show the time series behaviors of the different populations S_b^* , E_b^* , I_b^* , E_{vb}^* and I_{vb}^* . The values of the parameters are given in the test.

6. Discussion and Conclusion

We are interested in the transmission of a single serotype of dengue virus when two species of mosquitoes, *A. aegypti* and *A. albopictus* are co-circulating among a human population. The threshold number or basic reproduction of this model is defined as S_0 , where S_{0a} and S_{0b} are defined by equations (39a) and (39b). The square root of this number represents the average number of secondary cases that can cause when one initial infection is produced by a bite of one (either specie) mosquito. If $S_0 < 1$, the number of infections will go to 0 as time progresses. If however $S_0 > 1$, the number of infections will increase as time progress and the dengue virus infection will become endemic. The transmission of dengue virus depends on the values of $A_a(1 + \varepsilon_a \sin \tau(t + 194))$ and $A_b(1 + \varepsilon_b \sin \tau(t + 240))$. The behaviors of our solution depend on the value of ε_a and ε_b .

We have seen that the equilibrium points $S_{0a}^* = (S_a^*, E_a^*, I_a^*, E_{va}^*, I_{va}^*)$ and $S_{0b}^* = (S_b^*, E_b^*, I_b^*, E_{vb}^*, I_{vb}^*)$ when $\epsilon_a = 0$ and $\epsilon_b = 0$ are locally asymptotically stable when $S_0 > 1$. Figure 5 and Figure 8 show the behaviors of human and two species of vectors $(S_a^*, E_a^*, I_a^*, E_{va}^*, I_{va}^*)$ and $(S_b^*, E_b^*, I_b^*, E_{vb}^*, I_{vb}^*)$ as time passes.

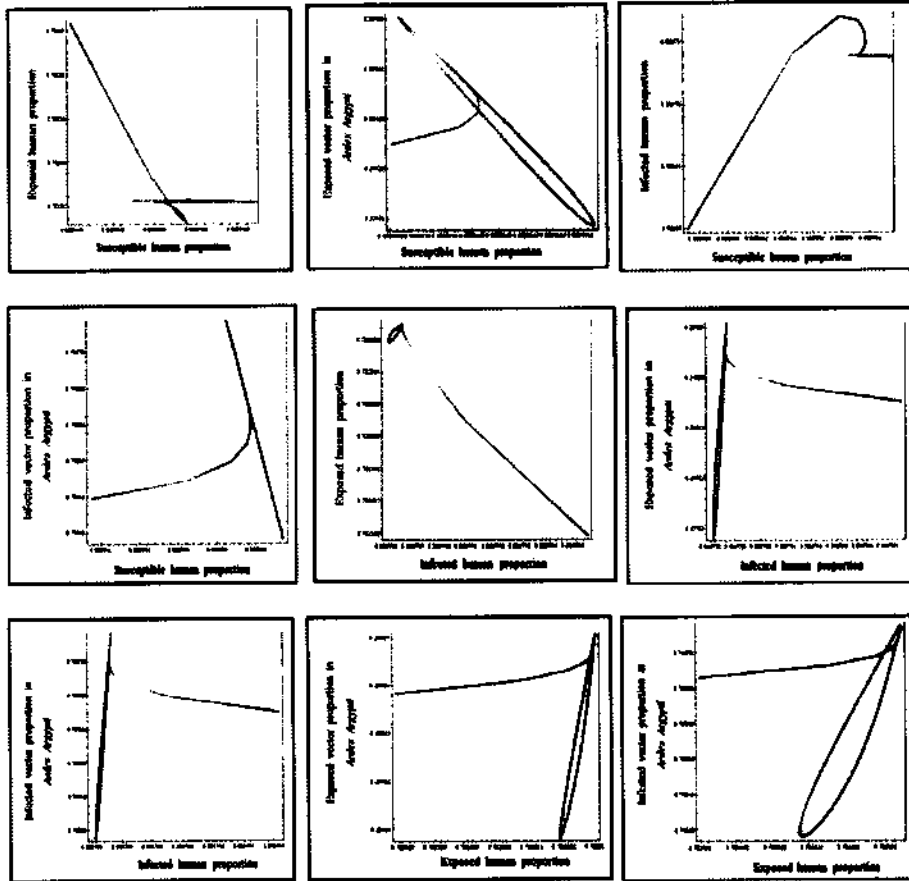


Figure 10. Limit cycle behavior of the populations in the model of dengue fever when the DENV is transmitted by *A. Albopictus* mosquitoes in different 2D space. The trajectories in the 2D space (S_b^*, E_b^*) , (S_b^*, E_{vb}^*) , (S_b^*, I_b^*) , (S_b^*, I_{vb}^*) , (I_b^*, E_b^*) , (I_b^*, E_{vb}^*) , (I_b^*, I_{vb}^*) , (E_b^*, E_{vb}^*) and (E_b^*, I_{vb}^*) exhibit limit cycle behaviors.

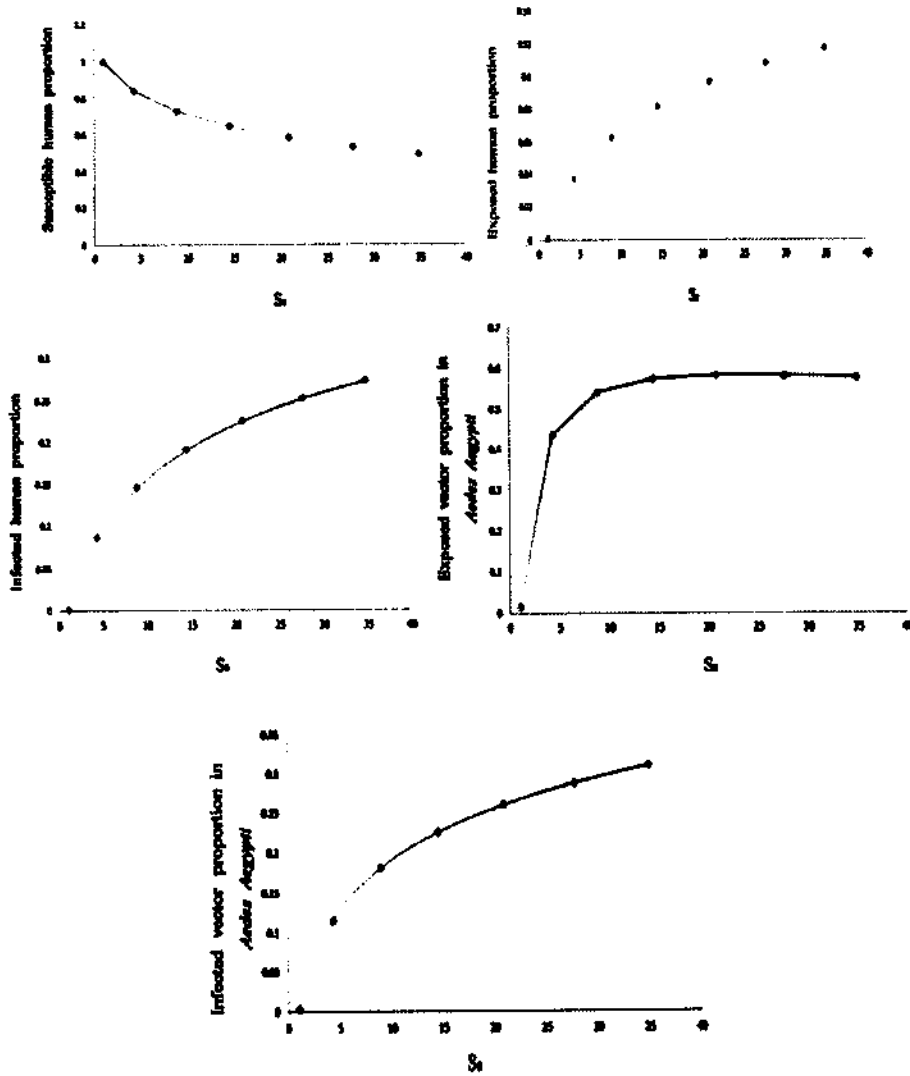


Figure 11. Bifurcation diagrams of the solutions to (16)-(26), plotted onto (S_0, S_a^*) , (S_0, E_a^*) , respectively for the different values of S_0 denote the stable solutions.

Each of the populations quickly reaches their equilibrium value and remains at the equilibrium values (Lemma). When $\epsilon_a \neq 0$ and $\epsilon_b \neq 0$, the endemic equilibrium points $S_{0a}^* = (S_a^*, E_a^*, I_a^*, E_{va}^*, I_{va}^*)$ and $S_{0b}^* = (S_b^*,$

E_b^* , I_b^* , E_{vb}^* , I_{vb}^*) are again locally stable when the other parameter values lead to $S_0 > 1$. The two dimensional plots however indicate another type of behavior (see Figures 6-7 and Figures 9-10). Instead of quickly coming to their equilibrium values, trajectories exhibit limit cycle behaviors (Figures 7 and 10). Varying the values of ϵ_a between 0.25 and 0.70 and ϵ_b between 0.3 and 0.65 for the cases of *A. aegypti* and *A. albopictus* vectors, respectively, we get behaviors similar to those of Figures 5 and 8 and those of Figures 6, 7, 9 and 10 when ϵ_a and ϵ_b are greater or less than some critical values (ϵ_a^* and ϵ_b^*) which we call the bifurcation values. When its value $\epsilon_a = 0$ and $\epsilon_b = 0$ or less than ϵ_a^* and ϵ_b^* , the time behaviors do not exhibit any oscillation, but if the values ϵ_a and ϵ_b are greater than ϵ_a^* and ϵ_b^* oscillations are seen. The bifurcation diagrams of equations (17)-(26) are shown in the above figures.

In conclusion, we showed that the endemic equilibrium point is local stable, when the threshold number is greater than one. The local stability of all equilibrium states are determined by the threshold numbers S_0 . This study shows that by modifying how one reacts to the environment so as to change the values of the parameters, it would be possible to control the spread of dengue fever when two species of mosquitoes, the *A. aegypti* and *A. albopictus* are co-circulating.

Acknowledgements

This work is supported by Faculty of Science and Technology Phuket Rajabhat University, King Mongkut's Institute of Technology Ladkrabang and National Research Council of Thailand.

The authors thank the anonymous referees for their valuable suggestions which led to the improvement of the manuscript.

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